Name:____ Date:____________

Section 6.2 Math 12 Honours Basics with Graphing Complex Numbers

1. Given that the modulus of a complex number is $r=\sqrt{a^2+b^2}$, can the modulus be negative? Explain:

2. Given that a complex number in polar form is $a+ib=r\left(\cos\theta+i\sin\theta\right)$, why is the argument θ only be between $-\pi \text{ and } \pi$? Explain:

3. If a complex number is in quadrant 3, what can you tell us about the argument? Explain:

4. What happens when you multiply a complex number "z" by just the imaginary value "i"? Explain:

5. Suppose the imaginary component of a complex number "z" is $\operatorname{Im}(z)=4\cos 17°$ and the modulus of "z' is 4, what the value of $\operatorname{Re}(z)=??$

6. Given that $\operatorname{Re}(z)=4\cos 30°$ and $\operatorname{Im}(z)=4\sin 30°$, what is the $\operatorname{Re}\left(z\times i^3\right)$ and $\operatorname{Im}\left(z\times i^3\right)$

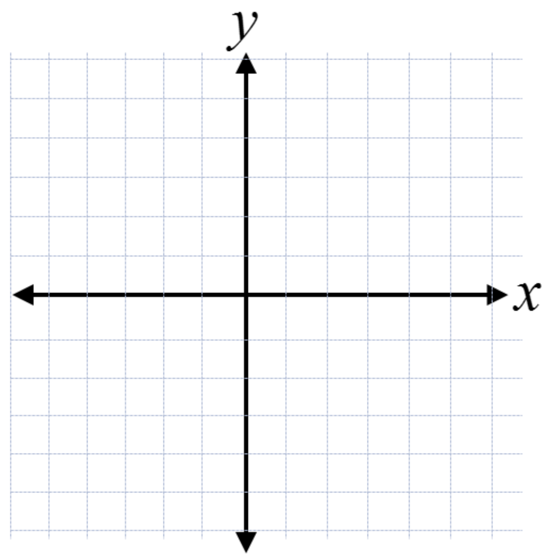
7. Are the two complex numbers the same? Explain:
   $z_1=\left(\cos 240°+i\sin 240°\right)^3$ and $z_2=\left(\cos\left(-120°\right)+i\sin\left(-120°\right)\right)^3$

8. Given that $z_1=\cos\theta_1+i\sin\theta_1$ and $z_2=\cos\theta_2+i\sin\theta_2$, what is $z_3=z_1\times z_2$ in polar form??

9. Given that $z_1=\cos\theta_1+i\sin\theta_1$ and $z_2=\cos\theta_2+i\sin\theta_2$, what is $z_3=z_1\div z_2$ in polar form??

10. Represent each of the following complex numbers on the Argand Diagram,

A) $3+i$	B) $-2+i$	C) $-3-i$	D) $1-2i$
E) $4i$	F) $-\sqrt{3}i$	G) $\sqrt{2}+4i$	H) i
I) $5-6i$	J) $-6+4i$	K) $2(\cos 45° + i\sin 45°)$	
L) $-2\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$		O) $-5\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$	



11. Find the Modulus and Argument for each of the complex numbers and then convert to polar form:

i) $\sqrt{3}+i$	ii) $-1-i\sqrt{3}$
iii) $12+5i$	iv) $-\sqrt{2}-\sqrt{2}i$
v) $6i$	vi) $-4i$
vii) $3+2i$	viii) $-3-3i\sqrt{3}$

ix) $(12+3i)(5-4i)$	x) $-2(\cos 30°+i\sin 30°)\times 3(\cos 45°-i\sin 45°)$

12. Reduce the following to rectangular form: $a+ib$

i) $4(\cos 90°+i\sin 90°)$	ii) $3(\cos 240°+i\sin 240°)$
iii) $2(\cos 315°+\sin 315°)$	iv) $-4\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)$
v) $-2\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)$	vi) $-5\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

13. Find $z_1\times z_2$, $\frac{z_1}{z_2}$, and also $\frac{z_2}{z_1}$: $z_1=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$ $z_2=\frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

14. Find $z_1 \times z_2$, $\frac{z_1}{z_2}$, and also $\frac{z_2}{z_1}$: given that : $z_1 = -2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ $z_2 = 3\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$

15. (1995 AIME) For certain real values a,b,c, and d, the equation $x^4 + ax^3 + bx^2 + cx + d = 0$ has four non-real roots. The product of two of these roots is $13+i$ and the sum of the other two roots is $3+4i$, where $i = \sqrt{-1}$. Find "b"

16. The polynomial $f(z) = az^{2018} + bz^{2017} + cz^{2016}$ has real coefficients not exceeding 2019, and $f\left(\frac{1+\sqrt{3}i}{2}\right) = 2015 + 2019\sqrt{3}i$. Find the values of "a", "b", and "c":

17.

(1988 AIME Problem 11) Let $w_1, w_2, \ldots, w_n$ be complex numbers. A line L in the complex plane is called a mean line for the points $w_1, w_2, \ldots, w_n$ if L contains points (complex numbers) $z_1, z_2, \ldots, z_n$ such that

$$\sum_{k=1}^{n}(z_k - w_k) = 0.$$

For the numbers $w_1 = 32 + 170i$, $w_2 = -7 + 64i$, $w_3 = -9 + 200i$, $w_4 = 1 + 27i$, and $w_5 = -14 + 43i$, there is a unique mean line with y-intercept 3. Find the slope of this mean line.